State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. A postulate is a statement that requires proof.

**SOLUTION:**
A postulate is a statement that does not require a proof. So, the sentence is false. A theorem is a statement that requires a proof.

2. The first part of an if-then statement is the conjecture.

**SOLUTION:**
The first part of an if-then statement is the hypothesis. So, the sentence is false.

3. Deductive reasoning uses the laws of mathematics to reach logical conclusions from given statements.

**SOLUTION:**
True

4. The contrapositive is formed by negating the hypothesis and conclusion of a conditional.

**SOLUTION:**
The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional. So, the sentence is false. The inverse is formed by negating the hypothesis and conclusion of a conditional.

5. A conjunction is formed by joining two or more statements with the word *and*.

**SOLUTION:**
True

6. A theorem is a statement that is accepted as true without proof.

**SOLUTION:**
A theorem is a statement that requires a proof. So, the sentence is false. A postulate is a statement that is accepted as true without proof.

7. The converse is formed by exchanging the hypothesis and conclusion of a conditional.

**SOLUTION:**
True

8. To show that a conjecture is false, you would provide a disjunction.

**SOLUTION:**
To show that a conjecture is false, you would provide a counterexample. So, the sentence is false.

9. The inverse of a statement $p$ would be written in the form $\neg p$.

**SOLUTION:**
The inverse is formed by negating both the hypothesis and conclusion of the conditional. So, the sentence is false. The negation of a statement $p$ would be written in the form $\neg p$.

10. In a two-column proof, the properties that justify each step are called *reasons*.

**SOLUTION:**
True

Determine whether each conjecture is true or false. If false, give a counterexample.

11. If $\angle 1$ and $\angle 2$ are supplementary angles, then $\angle 1$ and $\angle 2$ form a linear pair.

**SOLUTION:**
Two supplementary angles form a linear pair only if they share a common side. So, the sentence is false. Counterexample: Two nonadjacent supplementary angles.

12. If $W(-3, 2), X(-3, 7), Y(6, 7), Z(6, 2)$, then quadrilateral $WXYZ$ is a rectangle.

**SOLUTION:**
The sides $\overline{WZ}$ and $\overline{XY}$ are horizontal lines and $\overline{WX}$ and $\overline{YZ}$ are vertical lines. So, the quadrilateral has four right angles. Therefore, it is a rectangle, by definition.
13. **PARKS** Jacinto enjoys hiking with his dog in the forest at his local park. While on vacation in Smoky Mountain National Park in Tennessee, he was disappointed that dogs were not allowed on most hiking trails. Make a conjecture about why his local park and the national park have differing rules with regard to pets.

**SOLUTION:**
Sample answer: The national park may be home to wildlife species not found in the local park. Dogs or other pets may threaten or chase these unfamiliar animals or insects.

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain.

- **p**: A plane contains at least three noncollinear points.
- **q**: A square yard is equivalent to three square feet.
- **r**: The sum of the measures of two complementary angles is 180.

14. \( \sim q \lor r \)

**SOLUTION:**
Negate \( q \) finding the opposite truth values. Then find the disjunction \( \sim q \lor r \). A disjunction is true if at least one of the statements is true.
An a square yard is not equivalent to three square feet or the sum of the measures of two complementary angles is 180°. Here, \( \sim q \) is a true statement. Since one of the statements is true, the disjunction is also true.

15. \( p \land \sim r \)

**SOLUTION:**
Negate \( r \) finding the opposite truth values. Then find the conjunction \( p \land \sim r \). A conjunction is true only when both statements that form it are true.

A plane contains at least three noncollinear points is True. The sum of the measures of two complementary angles is not 180 is true. Since both the statements are true, the conjunction is also true.

16. \( \sim p \lor q \)

**SOLUTION:**
Negate \( p \) finding the opposite truth values. Then find the disjunction \( \sim p \lor q \). A disjunction is true if at least one of the statements is true.
The statement "A plane does not contain at least three noncollinear points" is false. The statement "a square yard is equivalent to three square feet" is false. Since both the statements are false, the disjunction is also false.

**Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.**

18. If you square an integer, then the result is a positive integer.

**SOLUTION:**
The conditional statement "If you square an integer, then the result is a positive integer." is true. When this hypothesis is true "you square an integer", the conclusion "the result is a positive integer" is also true, since the product of an integer multiplied by that same integer is always positive. So, the conditional statement is true.

19. If a hexagon has eight sides, then all of its angles will be obtuse.

**SOLUTION:**
A conditional with a false hypothesis is always true. Here, the hypothesis a hexagon has eight sides is false as a hexagon has six sides. Therefore, the conditional statement is true.
20. Write the converse, inverse, and contrapositive of the following true conditional. Then, determine whether each related conditional is true or false. If a statement is false, find a counterexample.

If two angles are congruent, then they have the same degree measure.

**SOLUTION:**

Converse: The converse is formed by exchanging the hypothesis and conclusion of the conditional. If two angles have the same degree measure, then they are congruent. The statement is true by the definition of congruence.

Inverse: The inverse is formed by negating both the hypothesis and conclusion of the conditional. If two angles are not congruent, then they do not have the same degree measure. The statement is true by the definition of congruence.

Contrapositive: The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional. If two angles do not have the same degree measure, then they are not congruent. The statement is true by the definition of congruence.

**Determine whether each statement is always, sometimes, or never true. Explain.**

24. Two planes intersect at a point.

**SOLUTION:**

If two planes intersect, they form a line. So, the statement "Two planes intersect at a point." is never true.

25. Three points are contained in more than one plane.

**SOLUTION:**

If the three points are non-collinear, then there exists a plane containing all the three points. But if the points are collinear, we can find three different planes with each plane containing one of the three points. So, the statement is sometimes true.

26. If line \( m \) lies in plane \( X \) and line \( m \) contains a point \( Q \), then point \( Q \) lies in plane \( X \).

**SOLUTION:**

If a plane contains a line, then every point of that line lies in the plane. Therefore, the statement "If line \( m \) lies in plane \( X \) and line \( m \) contains a point \( Q \), then point \( Q \) lies in plane \( X \)." is always true.

27. If two angles are complementary, then they form a right angle.

**SOLUTION:**

Two complementary angles form a right angle only if they are adjacent. Otherwise, they do not form a right angle. So, the statement is sometimes true.
State the property that justifies each statement.

29. If \(7(x - 3) = 35\), then \(35 = 7(x - 3)\).

**SOLUTION:**
The Symmetric Property of Equality is used to transform the equation \(7(x - 3) = 35\) to \(35 = 7(x - 3)\).

30. If \(2x + 19 = 27\), then \(2x = 8\).

**SOLUTION:**
Use the Subtraction Property of Equality to subtract 19 from each side of \(2x + 19 = 27\) to obtain \(2x = 8\).

31. \(5(x + 1) = 15x + 5\)

**SOLUTION:**
The Distributive Property to simplify \(5(x + 1)\) to \(15x + 5\).

32. \(7x - 2 = 7x - 2\)

**SOLUTION:**
The Reflexive Property of Equality describes \(7x - 2 = 7x - 2\).

33. If \(12 = 2x + 8\) and \(2x + 8 = 3y\), then \(12 = 3y\).

**SOLUTION:**
Use the Transitive Property of Equality to combine \(12 = 2x + 8\) and \(2x + 8 = 3y\) to \(12 = 3y\).

34. Copy and complete the following proof.

**Given:** \(6(x - 4) = 42\)

**Prove:** \(x = 3\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (6(x - 4) = 42)</td>
<td>a. ?</td>
</tr>
<tr>
<td>b. (6x - 24 = 42)</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. (6x = 66)</td>
<td>c. ?</td>
</tr>
<tr>
<td>d. (x = 11)</td>
<td>d. ?</td>
</tr>
</tbody>
</table>

**SOLUTION:**
The 1st contains the given information. The 2nd row uses the Distributive property to remove the parenthesis. The 3rd row uses addition to add 24 to each side. The 4th row uses division to divide each side by 11.

35. Write a two-column proof to show that if \(PQ = RS\), \(PQ = 5x + 9\), and \(RS = x - 31\), then \(x = -10\).

**SOLUTION:**
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two segments of equal length and expressions for each segment. Once you prove the values are equal, you will need to find the variable in the expression. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: \(PQ = RS\), \(PQ = 5x + 9\), and \(RS = x - 31\)

Prove: \(x = -10\)

**Proof:**

**Statements (Reasons)**
1. \(PQ = RS\), \(PQ = 5x + 9\), \(RS = x - 31\) (Given)
2. \(5x + 9 = x - 31\) (Substitution Property)
3. \(4x + 9 = -31\) (Subtraction Property)
4. \(4x = -40\) (Subtraction Property)
5. \(x = -10\) (Division Property)

**Write a two-column proof.**

37. Given: \(X\) is the midpoint of \(WY\) and \(YZ\)

**Prove:** \(VW = ZY\)

<table>
<thead>
<tr>
<th>(V)</th>
<th>(W)</th>
<th>(X)</th>
<th>(Y)</th>
<th>(Z)</th>
</tr>
</thead>
</table>

**SOLUTION:**
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the midpoint of two segments Use the properties that you have learned about congruent segments, midpoints, and equivalent expressions in algebra to walk through the proof.

Given: \(X\) is the midpoint of \(WY\) and \(YZ\)

**Prove:** \(VW = ZY\)

**Proof:**

**Statements (Reasons)**
1. \(X\) is the midpoint of \(WY\) and \(YZ\) (Given)
2. \(WX = YX\), \(VX = ZX\) (Definition of midpoint)
3. \(WX = YX\), \(VX = ZX\) (Definition of congruence)
4. \(VX = VW + WX\), \(ZX = ZY + YX\) (Segment. Addition Postulate.)
5. \(VW + WX = ZY + YX\) (Substitution)
6. \(VW = ZY\) (Subtraction Prop.)
38. **Given:** \( AB = DC \)
   **Prove:** \( AC = DB \)

   **Proof:**
   **Statements (Reasons)**
   1. \( AB = DC \) (Given)
   2. \( BC = BC \) (Reflexive Property)
   3. \( AB + BC = DC + BC \) (Addition Property)
   4. \( AB + BC = AC, DC + BC = DB \) (Segment Addition Property)
   5. \( AC = DB \) (Substitution)

**Find the measure of each angle.**

40. \( \angle 5 \)

   **SOLUTION:**
   Since the measure of the linear pair of \( \angle 5 \) is 90,
   \[ m\angle 5 = 180 - 90 = 90 \, ^\circ \].

41. \( \angle 6 \)

   **SOLUTION:**
   Angle 6 and the angle with measure 53 form a linear pair. So, \[ m\angle 6 = 180 - 53 = 127 \, ^\circ \].

42. \( \angle 7 \)

   **SOLUTION:**
   The \( \angle 7 \) and the angle with measure 53 are vertical angles. So, they are congruent.
   \[ m\angle 7 = 53 \]

43. **PROOF** Write a two-column proof.
   **Given:** \( \angle 1 \equiv \angle 4, \angle 2 \equiv \angle 3 \)
   **Prove:** \( \angle AFC \equiv \angle EFC \)

   **SOLUTION:**
   You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two sets of congruent angles. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.
   **Given:** \( \angle 1 \equiv \angle 4, \angle 2 \equiv \angle 3 \)
   **Prove:** \( \angle AFC \equiv \angle EFC \)
   **Proof:**
   **Statements (Reasons)**
   1. \( \angle 1 \equiv \angle 4, \angle 2 \equiv \angle 3 \) (Given)
   2. \( m\angle 1 = m\angle 4, m\angle 2 = m\angle 3 \) (Definition of congruence)
   3. \( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 \) (Addition Property)
   4. \( m\angle 1 + m\angle 2 = m\angle AFC, m\angle 3 + m\angle 4 = m\angle EFC \) (Angle Addition Postulate)
   5. \( m\angle AFC = m\angle EFC \) (Substitution)
   6. \( \angle AFC \equiv \angle EFC \) (Definition of congruence)